Proceedings

IEEE International Conference on Networks 2000

(ICON 2000)

Networking Trends and Challenges in the New Millennium

September 5–8, 2000

National University of Singapore, Singapore

IEEE

COMPUTER SOCIETY

Los Alamitos, California

Washington • Brussels • Tokyo

Session 7.2   Network Architectures and Protocols

Poster Session 1

Poster Session 2

Modelling Multi-Channel Slotted Ring Networks with Tunable Transmitters and Fixed Receivers

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Abstract

This study presents a non-preemptive priority queue model to approximate the cell delay of a multi-channel slotted ring network with tunable transmitters and fixed receivers, and one queue for storing cells for each channel at each node. To analyze network performance, this network is translated to the proposed non-preemptive priority queue model. With this model, the approximated analytical cell-delay results can be obtained by close-form formulas. This investigation assumes that the cell arrival rate for every destination at each node can be different and the number of nodes equals the number of channels. The accuracy of the model is assessed using the simulation results, and the two are found to be very close.

1. Introduction

Because of the limitation of electronic speed, the huge bandwidth of the fiber cannot be utilized efficiently. Recently, the WDM (Wavelength Division Multiplexing) [1] technology has been developed. It makes the bandwidth of the fiber can be used efficiently. By this technology, data can be transmitted concurrently in many channels on one fiber and each channel can send data at the electronic speed. There are many network topologies proposed for WDM networks. Among them, the WDM ring network is an efficient one to utilize the bandwidth of the fiber because the architecture and network interface of ring networks are simpler and it has the slot-reused property. The proposed node configurations of WDM ring networks can be separated into three kinds: multiple transmitters and receivers, single fixed transmitter and tunable receiver, and single tunable transmitter and fixed

receiver [2-5]. For the first case, its network utilization is best but its implementation cost is the highest among these configurations. Second, it has the capability to dynamically select channel to transmit cells but receiver collision might occur. Third, it is opposite to the second configuration. In this study, we discuss a network which nodes use the third configuration.

For WDM ring networks with tunable transmitters and fixed receivers, several transmission protocols are proposed in [5-6]. These protocols allow full connectivity through transmitter tunability and resolve the receiver collision problem by providing only one channel to every node for receiving cells. In these networks, there are also two different buffer strategies: one queue for all channels and one queue per one channel. In the first strategy, every node only has one queue to store cells awaiting transmission. In the second one, every node has one queue for cells awaiting transmission for every channel. So, in the second one, every node has queues which number is equal to the number of channels. With the first strategy, the network is simple but it has the head-of-the-line blocking problem [7]. The second strategy can avoid the head-of-the-line blocking but has more severe fairness problems than the first one [5]. In addition, the second one is also a basic buffer architecture to develop more complicate multichannel slotted ring protocols for providing reservation and fairness mechanisms, for example [8]. So the analyses of these strategies are valuable. This investigation proposes a non-preemptive priority queue model [9] to model the behavior of the second strategy in the multi-channel ring network with tunable transmitters and fixed receivers

There are many papers present the analytical models about slotted ring networks. In [2][10-14], the performance analytical models of single slotted ring network with source removal policy are presented. In [13-14], the analytical models with destination removal policy for single slotted ring network are also proposed. In

[2][13], the analytical models of multichannel slotted ring networks are presented. In these analytical models about multichannel slotted rings, the assumption that nodes are equipped with one transmitter and one receiver for each channel is adapted. The above analytical models are not appropriate to analyze the objective networks. In [7], the analytical model for the first buffer strategy, i.e. every node only has one queue for all channels, in the multichannel slotted ring networks with one tunable transmitter and one fixed receiver is presented.

This study utilizes the 'non-preemptive priority queue model' [9] for constructing the analytical model for multichannel slotted ring networks. In the network, every node has one queue for each channel to buffer these cells awaiting transmission. In this study, the multi-channel slotted ring network is successfully mapped into the queue model by treating every channel as a server- and transmission queues as priority queues. By this mapping, the delay analysis of the networks can obtain easily. Moreover, a simulation program is used to simulate the performance and cell delay of the network for assessing the accuracy of the analysis. The other portions of this paper consist of network architecture description and the presentation of transmission protocol of this multichannel slotted ring networks. In the cell-delay model section, the analytical model is described. Numerical results of the analysis are compared with the simulation results, and are shown at the end of this paper.

2. Network Architecture

The network architecture presented in this paper is a multiple channel ring system with time-slotted access. The number of channels W used in the network is same as the number of nodes M. Each node has one tunable transmitter and one fixed channel receiver. When a source node has data to transmit to a destination node, it must tune its transmitter to the channel on which the destination node is able to receive cells. Each node transmits fixed length cells that exactly fit into one slot. Each node is assigned a specified channel as its receiving channel to receive cells. Because the number of nodes is equal to that of channels, the network provides one logical channel for transmissions to each destination. Its logical behavior is the same as what is presented in [5-6].

The transmission protocol of this network is the RND MAC protocol proposed in [5] stated in the following. In every node, there is a individual queue for each transmission channel. When a node generates a cell, it puts the cell to the queue that is mapped to the receiving channel of the destination node of the cell. Then, when the cell is at head of the queue, it is attempted to transmit on its destination channel while the node selects the queue.

At every slot, each node randomly selects a queue from among its non-empty queues and attempt to transmit the head cell of the selected queue. If the destination channel is free, the transmission is successful, and the cell is removed from the queue. Otherwise, the cell is retained in the queue. When the destination node receives a cell on the receiving channel, it absorbs the cell and makes the slot empty. The difference between the original protocol and the modeled one is that the number of queues in each node is equal to the number of destination nodes in [5].

This multi-channel slotted ring network has a cyclic-priority property [5]. The property states that every node has better than average transmission capability to some channels and worse than average transmission capability to some others. The access capacity of every node constructs a priority list on every channel. As shown in Fig. 1, let the receiving channel of node i (i =0, 1, ..., 3) is channel i and the transmission direction is clockwise. From the above transmission protocol, node 1 has the best transmission capability than other nodes to node 0. Actually, node 1 can transmit its cell to node 0 at anytime because node 1 always detects that channel 0 is empty. On the contrary, node 1 has the worst transmission capability to node 2 because it can only use channel 2 when no other nodes transmit cells on the channel. Therefore, the cyclic-priority formula can be constructed. When $M=W$, the priority of node i is $(i-j+W)$ mod W for channel j(i, j=0, 1, ..., W-1), 1 being the highest and W-1 being the lowest priority.
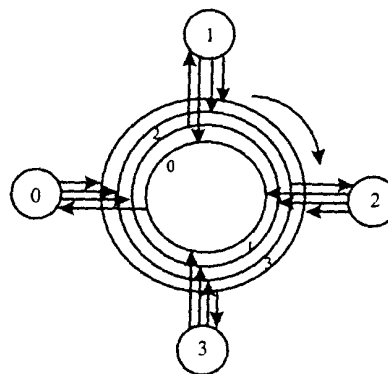
Fig. 1. Architecture of a multi-channel slotted ring network, when $M=W=4$.

3. Cell Delay

The cell delay of a cell is measured from the time the cell is completely stored in the queue of source node to when that cell is completely received by the destination node. This delay consists of queue-waiting delay, transmission delay and propagation delay. The queue-waiting delay of a cell is measured from when a cell is fully stored in a queue of the source node to the time the

source node last selected the queue before successful transmission. Meanwhile, in this ‘investigation, the transmission delay is defined as the interval between the source node selecting the queue to transmit the cell successfully and the time the source node last selected the queue before transmitting the cell successfully. Finally, the propagation delay of a cell is the interval between the time that the last bit of the cell reaches the destination and the moment the last bit of the cell was transmitted.

This work defines a special time period, the selected interval, to calculate the queue-waiting and transmission delays of a cell. The selected interval is that between two subsequent transmission attempts made by a node to transmit cells on the same channel. The definition of transmission delay indicates that the transmission delay of a cell is the selected interval at which the cell is transmitted successfully. Fig. 2 illustrates the timing diagram of a priority i node in a channel. The priority i node indicates that the transmission priority of the node on the channel is priority i. Every square in the figure represents a slot time at which the queue corresponding to the channel is selected by the node. In the figure, while assuming that the queue is empty when Cell 1 arrives, Cell 3 must wait for the transmission of Cells 1, 2 and of the cells of higher priority nodes represented by the blank square. Thus, the queue-waiting delay of Cell 3 comprises the residual time of the selected interval when it arrives a queue, that is the residual time of the selected interval when Cell 1 is transmitted successfully, and the selected intervals when Cell 2 and higher priority cells are transmitted before Cell 3. Generally, the queue-waiting delay of a cell from the priority i queue comprises the residual time of the selected interval at the arrival of the cell in the queue, the selected intervals when the node transmits cells that stay ahead of the cell in the queue successfully, and the selected intervals when the node fails to transmit queue cells successfully before this cell. Thus, in every selected interval a node transmits a maximum of one cell while the interval is the minimum interval between the transmission of two adjacent cells of a queue for every node.

From the above description, the operation of a priority i node in channel k can be represented by the queue model shown in Fig. 3 with i queues. This investigation assumes that for any given node the output rates of queues of other nodes equal their input rates, allowing the queue model to be mapped to the analyzed ring network. Because the status of a slot in a channel is detected only when a node selects its target queue, the output rate of higher priority nodes detected by the priority i node equals output cell rate of higher priority nodes divided by the selected interval length. Meanwhile, the slot server k in the figure represents the channel k. $Q_{(k+i)\oplus W,k}$ represents the queue of node $(k+i)\oplus W$ that corresponds to channel k. In the model, the slot server

takes one selected interval to service a cell. Meanwhile, the server checks all queues at every selected interval. When either $Q_{k+1,k}$, $Q_{k+2,k}$, $\cdots$, or $Q_{(k+i-1)\oplus W,k}$ attempts to transmit cells, these queues will take priority over $Q_{(k+i)\oplus W,k}$ for service. Thus, the cells of $Q_{(k+i)\oplus W,k}$ are served only when cells of higher priority queues do not request service.

□ : slot time at that priority i queue is selected
SI : selected interval
QD : queue-waiting delay
TD : transmission delay
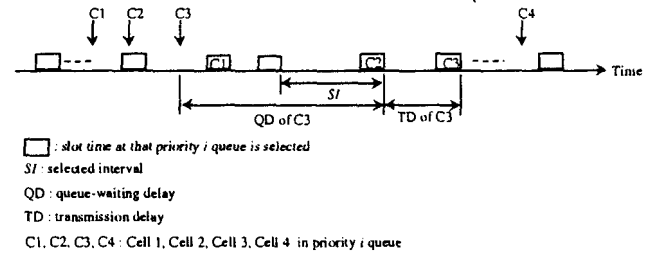C1, C2, C3, C4 : Cell 1, Cell 2, Cell 3, Cell 4 in priority i queue

Fig. 2. The timing diagram of a priority i node on one channel.

From the behavior of the queue model in Fig. 3, the model can be categorized as a non-preemptive priority queue model [9]. Intuitively, the mapped non-preemptive priority queue model views the selected interval for a queue as the time spent serving a cell because a cell of the queue or a cell from a higher priority queue is transmitted in a selected interval. For a queue of a node, the higher priority queues are those that corresponds to the same channel and are owned by higher priority nodes since the transmission of queue cells is delayed by the transmission of cells from higher priority nodes. Moreover, nodes can not preempt the slots that carry cells in slotted ring networks. Thus, for nodes with priority i in a channel, node behavior can be interpreted as the non-preemptive priority queue model with i priority classes on the channel. Therefore, finding the value of the selected interval and arrival rate of higher priority nodes in a channel, would allow the analytical results of a node with priority i in the channel to be modeled via the formulas of the non-preemptive priority queue model.
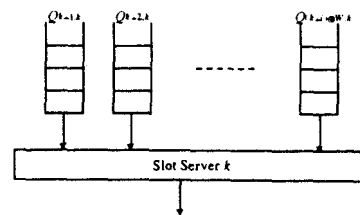

Fig. 3. Queue model of the corresponding queue of priority i node in channel k.

For simplicity, some assumptions are given as follows:

1. The number of nodes is equal to that of channels ($M=W$) and each node has its own receiving channel.

Let the receiving channel of node i (i =0, 1, ..., M-1) be channel i (i =0, 1, ..., W-1). Every node has W-1 queues, one for each transmission channel.

2. In the network, time is slotted. The propagation delay between neighboring nodes is d slot and so the propagation delay of the cell of node i on channel j is $((j-i+M) \bmod M)d$ slots.

3. The arrival rate is a Poisson distribution with rate $\lambda_{i,j}$ for node i on channel j. So, the total arrival rate to the network is $\sum_{i=0}^{M-1} \sum_{j=0, j \neq i}^{W-1} \lambda_{i,j}$.

4. Because the arrival rates of higher priority queues are Poisson distribution, for simplicity, we assume that the output rate of each higher priority queue is seen as Poisson distribution with its generated rate by priority i queue. But the assumption will make the slight difference between the analytical results and the simulation results.

Notations:

$Queue_{i,j}$: The queue of node i corresponding to channel j.

$P_{i,j} = (i-j+W) \bmod W$: The priority of node i on channel j.

$QD_{i,j}$: The average queue-waiting delay of cells of node i on channel j.

$\lambda_{i,j}$: The cell arrival rate of node i on channel j.

$\lambda_{i,j,k}$: The cell arrival rate of the node with priority k ($k=1...P_{i,j}-1$) inspected by the node i, on channel j.

$X_{i,j}$: The random variable representing the length of selected intervals of node i on channel j.

$\upsilon_{i,j}$: The number of the nonempty queues of node i when the queue corresponding to channel j is nonempty.

$\delta_{i,j}$: The probability that the slot is idle on channel j when node i attempts to transmit a cell on the channel.

$R_{i,j}$: Mean residual time of the selected interval when a new cell arrives at the queue of node i corresponding to channel j.

$T_{i,j}$: Mean cell delay of cells in $Queue_{i,j}$.

3.1 Cell Delay Analysis

Because the cell delay consists of queue-waiting delay, transmission delay and propagation delay, the average cell delay of $Queue_{i,j}$,

$$T_{i,j} = QD_{i,j} + E[X_{i,j}] + ((j-i+M) \bmod M)d, \quad (1)$$

and the average cell delay of the network,

$$T = \frac{\sum_{i=0}^{M-1} \sum_{j=1}^{W-1} T_{i,j}}{(M-1)(W-1)}. \quad (2)$$

From Appendix A, the average queue-waiting delay of cells in $Queue_{i,j}$ is

$$QD_{i,j} = \frac{R_{i,j}}{(1 - \sum_{k=1}^{P_{i,j}-1} \lambda_{i,j,k} E[X_{i,j}])} \times \frac{1}{(1 - \sum_{k=1}^{P_{i,j}-1} \lambda_{i,j,k} E[X_{i,j}] - \lambda_{i,j} E[X_{i,j}])} \quad (3)$$

When a cell arrives to a queue, the residual time of the cell is the residual time of current selected interval. From [4], the residual time can be written as

$$R_{i,j} = \max\left(\frac{E[X_{i,j}^2]}{2}\left(\sum_{k=1}^{P_{i,j}-1} \lambda_{i,j,k} + \lambda_{i,j}\right), \frac{1}{2}\right). \quad (4)$$

where the max(A,B) is a function that returns the larger value from among the two values. The left parameter of the function is the residual time of a cell in the non-preemptive priority queue model from [4] and the right one is the minimum average residual time of a cell in slotted ring network. Because in slotted ring network a slot time is the time unit, the minimum average residual time is a half slot time.

Because every node randomly selects a queue to transmit a cell from non-empty queues, the selection probability of $Queue_{i,j}$ is the reciprocal of $\upsilon_{i,j}$. Therefore, the first moment and second moment of a selected interval are

$$E[X_{i,j}] = \sum_{k=1}^{\infty} k\left(\frac{1}{\upsilon_{i,j}}\right)\left(1 - \frac{1}{\upsilon_{i,j}}\right)^{k-1} = \upsilon_{i,j}, \quad (5)$$

$$E[X_{i,j}^2] = \sum_{k=1}^{\infty} k^2\left(\frac{1}{\upsilon_{i,j}}\right)\left(1 - \frac{1}{\upsilon_{i,j}}\right)^{k-1}$$

$$= \frac{\left(2 - \frac{1}{\upsilon_{i,j}}\right)}{\left(\frac{1}{\upsilon_{i,j}}\right)^2} = \upsilon_{i,j}(2\upsilon_{i,j} - 1) \quad (6)$$

Because the slot status of channel j is detected only when $Queue_{i,j}$ is selected, the traffic rate of every higher priority node detected by node i is the cell rate generating by higher priority nodes divided by the selected interval length. Therefore, according to assumption 4,

$$\lambda_{i,j,k} = \frac{\lambda_{\omega(j,k),j}}{\upsilon_{i,j}}, \quad \omega(j,k) = (j+k) \bmod W, \quad k=1, \ldots,$$

$$P_{i,j} - 1. \quad (7)$$

Substituting $\lambda_{i,j,k}$, $R_{i,j}$ and $E[X_{i,j}]$ into Eq. (3), the average queue-waiting delay and the average cell delay of $Queue_{i,j}$ is written as

$$QD_{i,j} = \frac{2v_{i,j} - 1}{2(1 - \sum_{k=1}^{P_{i,j}-1} \lambda_{\omega(j,k),j})(1 - \sum_{k=1}^{P_{i,j}-1} \lambda_{\omega(j,k),j} - \lambda_{i,j}v_{i,j})}$$

$$T_{i,j} = \frac{2v_{i,j} - 1}{2(1 - \sum_{k=1}^{P_{i,j}-1} \lambda_{\omega(j,k),j})(1 - \sum_{k=1}^{P_{i,j}-1} \lambda_{\omega(j,k),j} - \lambda_{i,j}v_{i,j})}$$

$$+ v_{i,j} + ((j - i + M) \bmod M)d$$

3.2 Nonempty queue number

From assumption 4, the cell arrival rates of higher priority queues are their cell departure rates. Therefore, the probability that a slot is empty when the cell of $Queue_{i,j}$ is trying to transmit,

$$\delta_{i,j} = e^{-\sum_{k=1}^{P_{i,j}-1} \lambda_{\omega(j,k),j}}, \quad \omega(j,k) = (j + k) \bmod W . \quad (8)$$

where $\sum_{k=1}^{P_{i,j}-1} \lambda_{\omega(j,k),j}$ is the total output rate of higher priority queues. From (3) and (8), the probability that cells of $Queue_{i,j}$ are serviced at a slot can be obtained,

P[cell is serviced]=P[queue is selected]
    × P[slot is empty]

$$= \frac{1}{v_{i,j}} \delta_{i,j}. \quad (9)$$

Let the inter-cell-serviced time be the time between the time instant that neighboring cells of a queue are serviced. Then, the probability that the inter-cell-serviced time of cells of $Queue_{i,j}$ is k slots is $\frac{\delta_{i,j}}{v_{i,j}}(1 - \frac{\delta_{i,j}}{v_{i,j}})^{k-1}$

and the expected value of the inter-serviced time is $\frac{v_{i,j}}{\delta_{i,j}}$.

Because the probability that a queue is busy in a slot time is equal to the utilization of the queue, it is equal to multiplying arrival rate of cells and the inter-serviced time. Therefore, when $Queue_{i,j}$ is busy, the nonempty queue number is

$$v_{i,j} = \sum_{c=0,c\neq i,j}^{W-1} \lambda_{i,c} \frac{v_{i,c}}{\delta_{i,c}} + 1. \quad (10)$$

Similarly, the nonempty queue number when $Queue_{i,k}$ is busy can be obtained, that is

$$v_{i,k} = \sum_{c=0,c\neq i,k}^{W-1} \lambda_{i,c} \frac{v_{i,c}}{\delta_{i,c}} + 1. \quad (11)$$

Subtracting (10) from (11) yields

$$v_{i,k} = v_{i,j} - \frac{\lambda_{i,k}v_{i,k}}{\delta_{i,k}} + \frac{\lambda_{i,j}v_{i,j}}{\delta_{i,j}}. \quad (12)$$

Let $p_{i,j} = \frac{\lambda_{i,j}v_{i,j}}{\delta_{i,j}}$ and $b_{i,j} = \frac{\lambda_{i,j}}{\delta_{i,j}}$, then

$$v_{i,k} = v_{i,j} - p_{i,k} + p_{i,j}. \quad (13)$$

Substitute (13) into (10),

$$\therefore v_{i,j} = \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}(v_{i,j} - p_{i,c} + p_{i,j}) + 1. \quad (14)$$

From Appendix B, the (14) is deduced to

$$\therefore v_{i,j} = \frac{1}{1 - \sum_{c=0,c\neq i,j}^{W-1} (\frac{\lambda_{i,c}}{\delta_{i,c}} + \frac{\lambda_{i,c}(\lambda_{i,j}\delta_{i,c} - \lambda_{i,c}\delta_{i,j})}{(\lambda_{i,c} + \delta_{i,c})\delta_{i,j}\delta_{i,c}})} \quad (15)$$

4. Numerical Results

In this section, we will present the analytical and simulation results of cell delay of the network. The cell delay is the time spent by a cell in the system, that is, the difference between the time instant the last bit of the cell is received at the destination node and the time instant the cell enters a queue at the source node. In the evaluation model, the number of channels is same as that of nodes, $W=M=16$. The time is broken into slots. The distance between the neighboring nodes is 1 slot. The size of every queue is infinite. Two traffic scenarios, balanced traffic and unbalanced traffic, are considered in this evaluation. In the case of balanced traffic, all source nodes generate the same amount of traffic and destinations are equally likely. In the case of unbalanced traffic, one 'server' node generates an amount of traffic equal to half the total traffic generated by others nodes, and evenly distributes it to other nodes. The remaining M-1 'client' nodes direct half of their traffic to the server node, while the other half of the traffic is uniformly distributed among client nodes.

The analytical and simulation results of average cell delay in the multi-channel slotted ring network with balanced traffic are shown in Fig.4. These results show that the cell delay is small when the load of the network is light and its major portion is the cell propagation delay. In the heavy load, the cell delay grows up and its major portion will become the queue-waiting delay. The simulation results certify our analytical results, and the difference between them is made because of the assumption 4 in the analysis model.

In Fig.5, the cell delay of queues with different priority P is present. The figure shows that the cell delay of the highest priority queue is longer than the delay of other queues in light load, however its delay is better than the delay of other queues in heavy load. The reason is that the most portion of cell delay is the cell propagation delay

in the light load, however it will be the queue-waiting time in the heavy load. In addition, the figure shows that the network shows the cyclic-priority property.
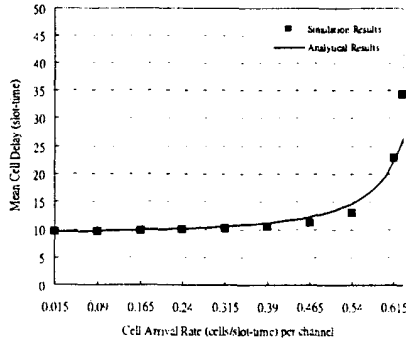

Figure 4. The mean cell delay for the WDM ring network with balanced traffic when M=W=16.
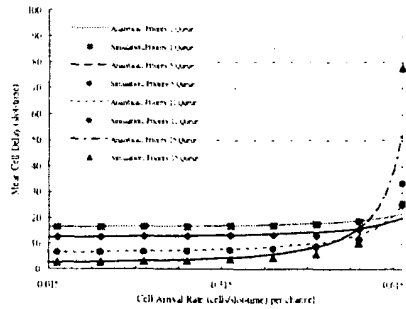

Figure 5. The mean cell delays of different priority queues under balanced traffic, when M=W=16 and priorities=1, 5, 11 and 15.

Figure 6 shows the cell delay of cells for all source-destination pairs when the traffic load per channel is 0.63 cell per slot time. The result is a simulative result. Herein, we want to prove the symmetry property of the network on balance traffic conditions by simulation results. The figure shows the symmetry on the cell delay. Every same priority queue has similar average cell delay. The slight difference between cell delay of same priority queues comes from simulation. That is said that these queues with same priority have the same cell delay on the balance traffic condition.

Fig. 7 and 8 show the analytical and simulation results for all source-destination pairs of the multi-channel slotted ring network under unbalanced traffic when the total traffic load is 2.5 cells per slot time, respectively. These results show the average cell wait-transmitted delay that consists of cell queue-waiting delay and cell transmission delay. The traffic load on the channel leading to the server is 0.83 and the load on these

channels leading to clients is 0.11. By comparing the two figures, it shows that the analytical approximate results nicely reproduce the behavior of simulation results. The reason for this inaccuracy of the analytical model is from the output rate assumption introduced in the model.
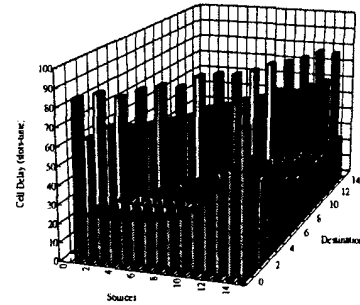

Figure 6. The simulation cell delay results for all source-destination pairs under balanced traffic, as M=W= 16.
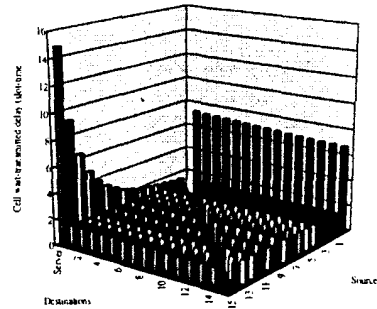

Fig. 7. The analytical cell wait-transmitted delay results for all source-destination pairs under unbalanced traffic, as M=W=16 and total traffic load=2.5.
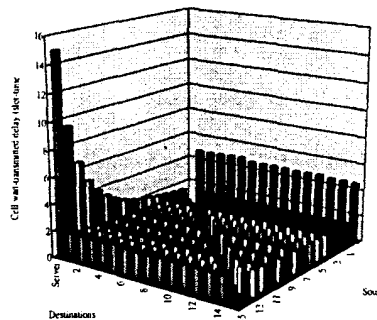

Fig. 8. The simulation cell wait-transmitted delay results for all source-destination pairs under unbalanced traffic, as M=W=16 and total traffic load=2.5.

5. Conclusions

In this paper, the approximated cell-delay analysis for the multi-channel slotted ring network with tunable transmitters and fixed receivers is presented. The equations for the cell delay of different priority queues are also derived. For verification, we also simulate the network using CACI Simscript II.5 and obtain the simulation results. It shows that the simulation results are approached the calculated values, and shows the performance and bottleneck of the network. These values can also be used for reference when the network is improved. Because of the importance of the WDM ring network, the research about the multi-channel slotted ring network is valuable.

Numerical results show that the RND MAC protocols achieves a reasonable bandwidth efficiency (63%), however, it has the bandwidth-sharing unfairness problem because of its cyclic-priority property. The unfairness problem can be overcome by fairness protocols, such as the Multi-MetaRing protocol [8]. The next topic of our research is to obtain the analysis in the condition that the number of nodes is not equal to that of channels..

6. Appendix

How to obtain the formula of the waiting delay in queue, $QD_{i,j}$, as W=M.

Because every node can transmit a maximum of one cell in a selected interval, from [9], the cell queueing delay of the highest priority node, node $j+1$, on channel j, $QD_{j+1,j}$ is

$$QD_{j+1,j} = R_{j+1,j} + N_{j+1,j}E[X_{j+1,j}].$$ (A1)

Here, $N_{j+1,j}$ is the number of cells in the queue corresponding to channel j in node $j+1$, and $R_{j+1,j}$ is the residual time inspected by the node. Meanwhile, $E[X]$ be the expected value of random variable X. By Little's theorem,

$$N_{j+1,j} = \lambda_{j+1,j}QD_{j+1,j}.$$ (A2)

$$\therefore QD_{j+1,j} = R_{j+1,j} + \lambda_{j+1,j}QD_{j+1,j}E[X_{j+1,j}]$$

$$= \frac{R_{j+1,j}}{1 - \lambda_{j+1,j}E[X_{j+1,j}]}.$$ (A3)

For the second priority node, node $j+2$, the traffic of highest priority node seen by the node on channel j, $\lambda_{j+2,j,1}$, must be considered. Its expression resembles (A3) except that the additional delay due to cells of highest priority node that arrival while a cell is queuing must be included.

$$QD_{j+2,j} = R_{j+2,j} + N_{j+2,j,1}E[X_{j+2,j}] + N_{j+2,j}E[X_{j+2,j}]$$
$$+ \lambda_{j+2,j,1} \cdot QD_{j+2,j}E[X_{j+2,j}]$$ (A4)

$N_{j+2,j,1}$ is the queue length of highest priority node detected by the second priority node. It differs from $N_{j+1,j}$ in (A1) because the traffic rate of highest priority node detected by the second priority node does not equal the traffic rate generated by highest priority node. The detected traffic rate is $\lambda_{j+2,j,1}$. Little's theorem obtains

$$QD_{j+2,j} = R_{j+2,j} + \lambda_{j+2,j,1}QD_{j+2,j,1}E[X_{j+2,j}]$$
$$+ \lambda_{j+2,j}QD_{j+2,j}E[X_{j+2,j}]$$
$$+ \lambda_{j+2,j,1}QD_{j+2,j}E[X_{j+2,j}]$$

$$QD_{j+2,j} = \frac{R_{j+2,j} + \lambda_{j+2,j,1}QD_{j+2,j,1}E[X_{j+2,j}]}{1 - \lambda_{j+2,j}E[X_{j+2,j}] - \lambda_{j+2,j,1}E[X_{j+2,j}]}.$$ (A5)

$QD_{j+2,j,1}$ is the queue-waiting delay of cells in highest priority node detected by the second priority node. It can be expressed in the form of $R_{j+2,j}, \lambda_{j+2,j,1}, E[X_{j+2,j}]$. Its derivation is similar to (A3) and

$$QD_{j+2,j,1} = \frac{R_{j+2,j}}{(1 - \lambda_{j+2,j,1}E[X_{j+2,j}])}.$$ (A6)

Then, the queue-waiting delay of cells in second priority node is

$$QD_{j+2,j} = \frac{R_{j+2,j}}{(1 - \lambda_{j+2,j,1}E[X_{j+2,j}])}$$
$$\times \frac{1}{(1 - \lambda_{j+2,j}E[X_{j+2,j}] - \lambda_{j+2,j,1}E[X_{j+2,j}])}.$$ (A7)

The derivation is similar for all node i on channel j. Because the cell output rate of every higher priority node is same as the arrival rate for node i under assumption 4, the formula of the waiting delay in queue is

$$QD_{i,j} = \frac{R_{i,j}}{(1 - \sum_{k=1}^{P_{i,j}-1}\lambda_{i,j,k}E[X_{i,j}])}$$
$$\times \frac{1}{(1 - \sum_{k=1}^{P_{i,j}-1}\lambda_{i,j,k}E[X_{i,j}] - \lambda_{i,j}E[X_{i,j}])}.$$ (A8)

How to deduce the formula (15) from (14).

In (14), υ_i is expressed by

$$\upsilon_{i,j} = \sum_{c=0,c\neq i,j}^{W-1}b_{i,c}(\upsilon_{i,j} - p_{i,c} + p_{i,j}) + 1.$$ (B1)

$$\therefore \upsilon_{i,j} = \left(\sum_{c=0,c\neq i,j}^{W-1}b_{i,c}\right)\upsilon_{i,j} - \sum_{c=0,c\neq i,j}^{W-1}b_{i,c}(p_{i,c} - p_{i,j}) + 1.$$ (B2)

Move the right hand part of $v_{i,j}$ to the left hand, the formula is expressed by

$$(1 - \sum_{c=0,c\neq i,j}^{W-1} b_{i,c})v_{i,j} = \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}(p_{i,j} - p_{i,c}) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}(b_{i,j}v_{i,j} - b_{i,c}v_{i,c}) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}(b_{i,j}v_{i,j} - b_{i,c}(v_{i,j} - p_{i,c} + p_{i,j})) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}((b_{i,j} - b_{i,c})v_{i,j} - b_{i,c}(p_{i,j} - p_{i,c})) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}((b_{i,j} - b_{i,c})v_{i,j} - b_{i,c} \cdot (b_{i,j}v_{i,j} - b_{i,c}v_{i,c})) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}((b_{i,j} - b_{i,c})v_{i,j} - b_{i,c}(b_{i,j} - b_{i,c})v_{i,j} + b_{i,c}^2((b_{i,j} - b_{i,c})v_{i,j} - b_{i,c}(p_{i,j} - p_{i,c}))) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}((b_{i,j} - b_{i,c})v_{i,j} - b_{i,c}(b_{i,j} - b_{i,c})v_{i,j} + b_{i,c}^2(b_{i,j} - b_{i,c})v_{i,j} - b_{i,c}^3(p_{i,j} - p_{i,c})) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}((b_{i,j} - b_{i,c})v_{i,j} - b_{i,c}(b_{i,j} - b_{i,c})v_{i,j} + b_{i,c}^2(b_{i,j} - b_{i,c})v_{i,j} - b_{i,c}^3(b_{i,j} - b_{i,c})v_{i,j} + \cdots) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}(v_{i,j}(b_{i,j} - b_{i,c}) \times (\sum_{s=0}^{\infty}(-b_{i,c})^s)) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} b_{i,c}(v_{i,j}(b_{i,j} - b_{i,c})\frac{1}{b_{i,c} + 1}) + 1$$

$$= \sum_{c=0,c\neq i,j}^{W-1} \frac{b_{i,c}}{b_{i,c} + 1}(b_{i,j} - b_{i,c})v_{i,j} + 1$$

(B3)

$$\therefore v_{i,j} = \frac{1}{1 - \sum_{c=0,c\neq i,j}^{W-1} (\frac{\lambda_{i,c}}{\delta_{i,c}} + \frac{b_{i,c}(b_{i,j} - b_{i,c})}{b_{i,c} + 1})}$$

$$= \frac{1}{1 - \sum_{c=0,c\neq i,j}^{W-1} (\frac{\lambda_{i,c}}{\delta_{i,c}} + \frac{\lambda_{i,c}(\lambda_{i,j}\delta_{i,c} - \lambda_{i,c}\delta_{i,j})}{(\lambda_{i,c} + \delta_{i,c})\delta_{i,j}\delta_{i,c}})} \quad (B4)$$

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