

Performance evaluation of multiple-channel slotted ring networks with tunable transmitters and fixed receivers

Jun-Yao Wang^{a,*}, Wen-Shyang Hwang^b, Wen-Fong Wang^a, Ce-Kuen Shieh^a

^aDepartment of Electrical Engineering, National Cheng-Kung University, No. 1 Ta Hsueh Road, Tainan 7001, Taiwan, ROC

^bDepartment of Electrical Engineering, National Kaohsiung Institute of Technology, Kaohsiung, Taiwan, ROC

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Abstract

This study presents a non-preemptive priority queue model to approximate the cell delay of a multi-channel slotted ring network with a single tunable transmitter and fixed receiver, and one queue for storing cells for each channel at each node. To analyze network performance, this network is translated to the proposed non-preemptive priority queue model. With this model, the analytical cell-delay approximations can be obtained by close-form formulas. The analysis considers two network environments: where the number of nodes equals the number of channels and the number of nodes is a multiple of the number of channels. The accuracy of the model is assessed using the simulation results, and the two are found to be very close. In addition to its appropriateness for the multi-channel slotted ring network, the proposed analytical model can also derive the cell-delay results of a single-channel slotted network with the destination removal policy. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Multiple-channel slotted ring network; Non-preemptive priority queue model; Wavelength division multiplexing

1. Introduction

Wavelength division multiplexing (WDM) [1–4], first developed during the late-1980s, provides means of building multiple channels over slotted ring networks. A multi-channel slotted ring network can thus provide tremendous bandwidth, with total bandwidth increasing to hundreds or thousands of times the bandwidth of a single ring network without WDM. The proposed node configurations for multi-channel ring networks can be classified into three types: multiple transmitters and receivers, one fixed transmitter and one tunable receiver, and one tunable transmitter and one fixed receiver [5]. The first type has the best network utilization but is the most costly. Meanwhile, although the second type can dynamically select channels, receiver collisions are possible. Finally, the third type resembles the second in media access except for swapping the tunability of the receiver for that of the transmitter. All the node configurations with appropriate medium access control (MAC) protocols can provide full communication between nodes. This investigation focuses on the third node configuration to consider implementation feasibility. In this

configuration, each destination node is assigned the transmission channel, and the transmitter has complete tunability over all channels. The configuration provides the full connection through transmitter tunability and overcomes the receiver contention problem by assigning only one channel to each destination node.

To avoid transmission collisions among nodes attempting to transmit to the same destination, a MAC protocol is necessary to arbitrate access to the shared channel. Several MAC protocols have been proposed for WDM ring networks with a single tunable transmitter and a fixed receiver, for example, FIFO, RND, ARR, SRR protocols [6–8]. Based on the number of queues at each node, these MAC protocols can be separated into those with one queue for all channels at each node like FIFO, and those with a queue for each channel at one node such as RND, ARR, and SRR. Meanwhile, the FIFO protocol is the simplest, with every node only having one logical first-in-first-out (FIFO) queue for packets awaiting transmission. However, this protocol incurs the head of line (HOL) problem [6]. On the other hand, the remaining protocols can avoid the HOL problem and provide better performance owing to every node maintaining separate queues for different channels or destinations. To enhance network utilization, these protocols adopt the destination removal policy, that is, allowing destination nodes to remove cells from the incoming slots. To

* Corresponding author. Tel.: + 886-62-757575; fax: + 886-62-748678.

E-mail address: wjy@eembox.ee.ncku.edu.tw (J.-Y. Wang).

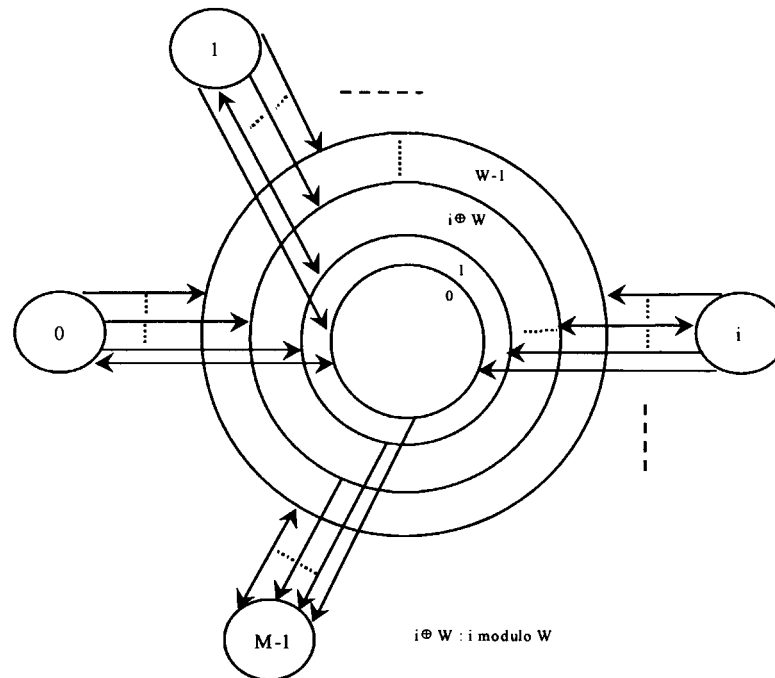


Fig. 1. Architecture of a multi-channel slotted ring network, where M is a multiple of W .

simplify analysis, this study investigates the performance of the simplest of the latter protocols, the RND protocol. This investigation applies a non-preemptive priority queue model to mimic the behavior of nodes in the networks and provides the analytical cell-delay results.

Many studies (e.g. Zafirovic-Vukotic et al. [9], Lee et al. [10], Bux [11] and Bhuyan et al. [12]) have developed analytical models for single channel slotted ring networks. Kang et al. [13] proposed and analyzed a multi-channel slotted ring network with multiple transmitters and receivers (every node has a transmitter and a receiver for each channel). Their analytical model uses a probability model to approximate the cell delay of the network. Notably, for a multi-channel slotted ring network with a tunable transmitter and fixed receiver, Marsan et al. [14] presented an approximate analytical model based on the FIFO protocol. Assuming an infinite-buffer queue, this approach models the behavior of a packet in the transmission queue at each node using a discrete-time M/G/1 queue. Their study provides the simulation throughput and analytical cell delay of the network. Additionally, Marson et al. [6] presented the performance of the RND protocol. However, their study only provides the throughput results. Our investigation models the multi-channel slotted ring network with the RND protocol based on the non-preemptive priority queue model [15]. Because the slotted ring networks which apply destination removal policy possess the cyclic-priority property described later in this investigation, the transmission queues of every node and channel could be taken as priority queues and servers, respectively. Finally, this study can give the analytical results for performing RND, including the approximate average cell-delay.

To demonstrate the accuracy of the analytical model, this work simulates the performance of the network, including the average cell delay. Additionally, the network architecture and RND for a multi-channel slotted ring network are presented. The rest of this paper is organized as follows. The cell-delay model (Section 3) describes how to translate the network into the non-preemptive priority queue model, and it also includes a few assumptions and special notes. Meanwhile, Sections 4 and 5 present the analytical formulas for two architectural cases: where the number of nodes equals the number of channels, and where the number of nodes is a multiple of the number of channels. The numerical analytical results are compared with the simulation results, and presented in Section 6. Concluding remarks are finally made in Section 7.

2. Network architecture

The analyzed network is a multiple channel slotted ring network with M nodes and W channels, as illustrated in Fig. 1. The W channels run in parallel. Every channel in the network is separated into several fixed slots and these slots move synchronously and unidirectionally through the ring. Every transmitted cell fits into the payload of a slot.

Each node has a single tunable transmitter and fixed receiver, and is assigned a channel to receive cells. Thus, when a node attempts to transmit cells to its destination node, it transmits its cells on the assigned channel of the destination node. The transmitter is assumed to be tunable on a slot-by-slot basis, thus allowing slotted access. When a node endeavors to transmit a cell, its transmitter tunes to the

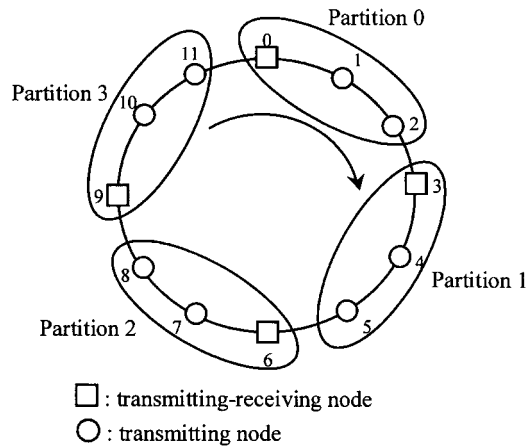


Fig. 2. Node distribution in one channel of a multi-channel slotted ring network, when $W = 3$ and $M = 12$.

assigned channel of the destination node. Then, when receiving the cell through its fixed receiver, the destination node removes the cell from the slot.

In the network, a node has one queue corresponding to each channel. When attempting to transmit a cell through a channel, a node stores the cell in the queue which relates to the assigned channel of the destination node. At every slot time, each node randomly selects one queue from these non-empty queues and then tries to transmit the first cell in the queue at the next slot time. If a node attempts to transmit a cell when the slot in the destination channel is empty, the cell is successfully transmitted. However, if the slot is full, the cell continues queuing until the next opportunity.

While assuming balanced traffic distribution, nodes are distributed evenly among channels. If M is equal to W , the assigned channel of node i is channel i , $0 \leq i \leq M$. Meanwhile, if M is larger than W , the node traffic load is assumed to be distributed evenly among channels, thus utilizing the network most efficiently given a balanced traffic condition. This implies that nodes are assigned their channel fairly, and are equally spaced on their assigned channels. If k is equal to M/W , k nodes are sharing one channel to receive cells in the network. Node i is assigned the channel $j = i \oplus W$ ($i \oplus W$ means i modulo W), $0 \leq j < W$. These assumptions separate all nodes into k partitions for one channel and every partition has W nodes. Fig. 2 presents the node distribution example of $M > W$ on a channel. In the example, the network contains 12 nodes and 3 channels. Therefore, $k = 12/3 = 4$ nodes sharing a channel and $W = 3$ nodes exist per partition. In every partition, one node can transmit and receive cells, the *transmitting-receiving node*; and $k - 1$ nodes can transmit cells only, the *transmitting nodes*. Transmitting-receiving nodes use their assigned channel to send cells to transmitting-receiving nodes in other partitions and receive cells from other nodes. Meanwhile, transmitting nodes can only transmit cells to transmitting-receiving nodes through the channel.

In multi-channel slotted ring networks, the destination

removal policy is applied to enhance network throughput. Since multi-channel slotted ring networks are ring-style networks, node location implies a cyclic-priority property [6]. This property indicates that a node has above average access capability for some channels and below average access capability for other channels. For example, Fig. 1 assumes that W is equal to M and the assigned channel of node i is channel i ($i = 0, 1, \dots, M - 1$). Also, the network transmits in a clockwise direction. In the topology, node 1 has the best access capability for channel 0 because it always finds channel 0 empty. Meanwhile, node 1 has the worst access for channel 2 because it must wait for other nodes to transmit cells through the channel before using it.

When M equals W , a priority formula can be developed. The access priority of node j is $(j - i + W) \oplus W$ ($i \oplus W$ means i modulo W) in channel i ($i = 0, 1, \dots, W - 1, j = 0, 1, \dots, M - 1$), where 1 represents the highest priority and $W - 1$ the lowest priority. When M equals $k \times W$ (k is an integer and larger than 1), the node distribution of each partition is identical on all channels given the above way of channel assignment. Since slots can be reused, the cell transmission of a transmitting-receiving node is not deferred by other nodes in the same partition. Therefore, transmitting-receiving nodes have the highest priority in the partition, while other nodes in the partition have lower transmission priorities due to their channel position. Furthermore, a transmitting-receiving node may defer its transmission due to the transmission of nodes in other partitions because several nodes are sharing a single assigned channel. This differs from the priority 1 node in the case of $k = 1$. Here, to interpret the access opportunity on a channel, the priority of the transmitting-receiving node is fixed as 0 in a partition, and the priorities of other nodes are set from 1 to $W - 1$ according to their distance from the transmitting-receiving node.

3. The cell-delay model

The cell delay of a cell is measured from the time the cell is completely stored in the queue of source node to when that cell is completely received by the destination node. This delay consists of queue-waiting delay, transmission delay and propagation delay. The queue-waiting delay of a cell is measured from when a cell is fully stored in a queue of the source node to the time the source node last selected the queue before successful transmission. Meanwhile, in this investigation, the transmission delay is defined as the interval between the source node selecting the queue to transmit the cell successfully and the time the source node last selected the queue before transmitting the cell successfully. Finally, the propagation delay of a cell is the interval between the time that the last bit of the cell reaches the destination and the moment the last bit of the cell was transmitted.

This work defines a special time period, the selected

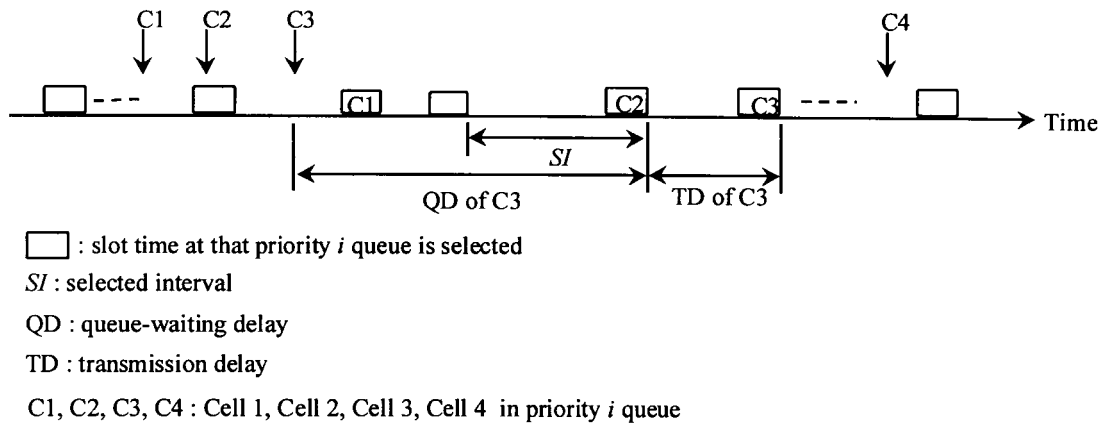


Fig. 3. The timing diagram of a priority i node on one channel.

interval, to calculate the queue-waiting and transmission delays of a cell. The selected interval is that between two subsequent transmission attempts made by a node to transmit cells on the same channel. The definition of transmission delay indicates that the transmission delay of a cell is the selected interval at which the cell is transmitted successfully. Fig. 3 illustrates the timing diagram of a priority i node in a channel. The priority i node indicates that the transmission priority of the node on the channel is priority i . Every square in the figure represents a slot-time at which the queue corresponding to the channel is selected by the node. In the figure, while assuming that the queue is empty when Cell 1 arrives, Cell 3 must wait for the transmission of Cells 1, 2 and of the cells of higher priority nodes represented by the blank square. Thus, the queue-waiting delay of Cell 3 comprises the residual time of the selected interval when it arrives a queue, that is the residual time of the selected interval when Cell 1 is transmitted successfully, and the selected intervals when Cell 2 and higher priority cells are transmitted before Cell 3. Generally, the queue-waiting delay of a cell from the priority i queue comprises the residual time of the selected interval at the arrival of the cell in the queue, the selected intervals when the node transmits cells that stay ahead of the cell in the queue successfully, and the selected intervals when the node fails to transmit queue cells successfully before this cell. Thus, in every selected interval a node transmits a maximum of one cell while the interval is the minimum interval between the transmission of two adjacent cells of a queue for every node.

From the above description, when M equals W , the operation of a priority i node in channel k can be represented by the queue model shown in Fig. 4 with i queues. This investigation assumes that for any given node the output rates of queues of other nodes equal their input rates, allowing the queue model to be mapped to the analyzed ring network. Because the status of a slot in a channel is detected only when a node selects its target queue, the output rate of higher priority nodes detected by the priority i node equals output cell rate of higher priority nodes divided by the selected interval length. Meanwhile, the slot server k in

the figure represents the channel k . $Q_{(k+i) \oplus W, k}$ represents the queue of node $(k+i) \oplus W$ that corresponds to channel k . In the model, the slot server takes one selected interval to service a cell. Meanwhile, the server checks all queues at every selected interval. When either $Q_{k+1, k}$, $Q_{k+2, k}$, ..., or $Q_{(k+i-1) \oplus W, k}$ attempts to transmit cells, these queues will take priority over $Q_{(k+i) \oplus W, k}$ for service. Thus, the cells of $Q_{(k+i) \oplus W, k}$ are served only when cells of higher priority queues do not request service.

From the behavior of the queue model in Fig. 4, the model can be categorized as a non-preemptive priority queue model [15]. Intuitively, the mapped non-preemptive priority queue model views the selected interval for a queue as the time spent serving a cell because a cell of the queue or a cell from a higher priority queue is transmitted in a selected interval. For a queue of a node, the higher priority queues are those that corresponds to the same channel and are owned by higher priority nodes since the transmission of queue cells is delayed by the transmission of cells from higher priority nodes. Moreover, nodes cannot preempt the slots that carry cells in slotted ring networks. Thus, for nodes with priority i in a channel, node behavior can be interpreted as the non-preemptive priority queue model with i priority classes on the channel. Therefore, finding

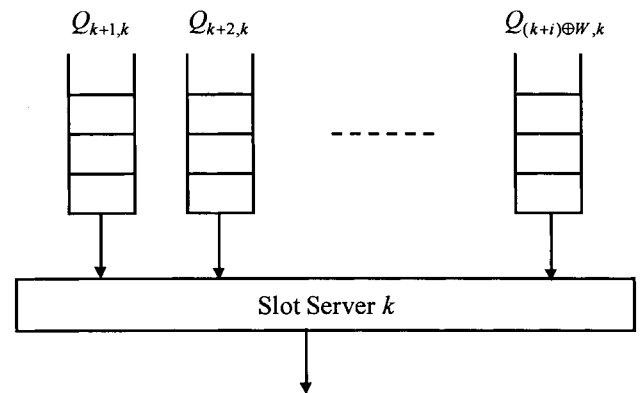


Fig. 4. Queue model of the corresponding queue of priority i node in channel k , when $M = W$.

the value of the selected interval and arrival rate of higher priority nodes in a channel, would allow the analytical results of a node with priority i in the channel to be modeled via the formulas of the non-preemptive priority queue model.

Herein, if the priority of a node in a channel is i , the queue of the node corresponding to the channel is termed priority i ($i = 0, \dots, W - 1$) queue. Meanwhile, the node with priority i in a channel is called the priority i node in the channel. The subsequent section analyzes the cell delay of the priority i node.

Assumptions. For simplicity, the following assumptions are made:

1. The number of nodes is a multiple of the number of channels ($M = Wk$, k is an integer and $k \geq 1$), each node has its own assigned channel, the assigned channel of node i ($i = 0, 1, \dots, M - 1$) is channel $i \oplus W$, and each node has W queues, one for each transmission channel.
2. Time in the network is slotted, and the propagation delay between neighboring nodes is d slots.
3. The cell arrival rate at each node is identical, and a cell is distributed to $M - 1$ destination nodes with equal probability. Meanwhile, at each node, the arrival rate is a Poisson distribution with a rate of λ for every destination. Therefore, the total arrival rate for the network is $M(M - 1)\lambda$. For a channel, the traffic load provided by a transmitting-receiving node is $(k - 1)\lambda$, and that provided by a transmitting node is $k\lambda$.
4. Because the arrival rates of higher priority queues have a Poisson distribution, for simplicity, this investigation assumes that the output rate for every destination at each higher priority node also has Poisson distribution with rate λ for priority i node. Since the output rate is not exactly a Poisson distribution, this assumption will cause slight discrepancy between analytical results and simulation results.

Notations. The following notations are used in the analytical formulas below:

TQ_i	the average queue-waiting delay of priority i queue cells
λ	the cell arrival rate for every destination at each node
λ'_i	the cell output rate for every destination at each higher priority node inspected by priority i node on a channel
X_i	the random variable representing the length of selected intervals
v_i	the number of non-empty queues of a node whose priority i queue is not empty
δ_i	the probability that the slot is empty in the channel on which a node attempts to transmit a cell of its own priority i queue
R_i	mean residual time of the selected interval when a new cell arrives a priority i queue

T_i	mean cell delay of priority i queue cells
T_{avg}	mean cell delay in the network.

4. Analysis of the equivalent case

This section presents the analytical method concerning the case of $k = 1$. When k is 1, the number of nodes equals the number of channels, i.e. $M = W$. Meanwhile, in the model, every node has $W - 1$ queues to store cells that are attempting to transmit to the network. Therefore, every node has queues with priorities from 1 to $W - 1$.

Because the cell delay comprises queue-waiting delay, transmission delay and propagation delay, the average cell delay of priority i queue is

$$T_i = TQ_i + E[X_i] + (M - 1 - i)d \tag{1}$$

and the average cell delay of the network is

$$T_{\text{avg}} = \frac{\sum_{i=1}^{W-1} T_i}{W - 1} \tag{2}$$

From Appendix A, the average queue-waiting delay of cells in priority i queue is

$$TQ_i = \frac{R_i}{\left(1 - \sum_{k=1}^{i-1} \lambda'_k E[X_k]\right) \left(1 - \sum_{k=1}^{i-1} \lambda'_k E[X_k] - \lambda E[X_i]\right)} \tag{3}$$

When a cell arrives at a queue, the residual time of the cell is the residual time of the currently selected interval. From Ref. [15], the residual time can be written as

$$R_i = \frac{1}{2} \frac{E[X_i^2]}{E[X_i]} \tag{4}$$

Because every node randomly selects a queue to transmit a cell from non-empty queues, the selection probability of the priority i queue is the reciprocal of v_i . Therefore, the first and second moment of a selected interval is obtained by

$$E[X_i] = \sum_{k=1}^{\infty} k \left(\frac{1}{v_i}\right) \left(1 - \frac{1}{v_i}\right)^{k-1} = v_i \tag{5}$$

$$\begin{aligned} E[X_i^2] &= \sum_{k=1}^{\infty} k^2 \left(\frac{1}{v_i}\right) \left(1 - \frac{1}{v_i}\right)^{k-1} = \frac{\left(2 - \frac{1}{v_i}\right)}{\left(\frac{1}{v_i}\right)^2} \\ &= v_i(2v_i - 1) \end{aligned} \tag{6}$$

Because the status of a slot on the channel is discovered only once the priority i queue is selected, the traffic rates of higher priority nodes discovered by the priority i node are the cell rates generated by higher priority nodes divided by the selected interval length. Therefore, according to

assumption 4,

$$\lambda'_i = \frac{\lambda}{v_i} \quad (7)$$

Substituting λ'_i , R_i and $E[X_i]$ into Eq. (3), the average queue-waiting delay and the average cell delay of the priority i queue is written as

$$TQ_i = \frac{2v_i - 1}{2(1 - (i - 1)\lambda)(1 - (i - 1)\lambda - \lambda v_i)}$$

$$T_i = \frac{2v_i - 1}{2(1 - (i - 1)\lambda)(1 - (i - 1)\lambda - \lambda v_i) + v_i + (M - 1 - i)d}$$

4.1. Non-empty queue number for $k = 1$

From assumption 4, the cell-arrival rates of higher priority queues are their cell-departure rates. Therefore, the probability that a slot is empty when the cell of priority i queue is trying to transmit

$$\delta_i = e^{-\sum_{j=1}^{i-1} \lambda} \quad (8)$$

where $\sum_{j=1}^{i-1} \lambda$ is the total output rate of higher priority queues under assumption 4. From Eqs. (5) and (8), the probability that cells of priority i queue are serviced at a slot can be obtained

$$P[\text{cell is serviced}] = P[\text{queue is selected}] \times P[\text{slot is empty}] = \frac{1}{v_i} \delta_i. \quad (9)$$

Let the inter-cell serviced time be the time between moments when neighboring cells of the queue are serviced. Then, the probability that the inter-cell-serviced time of cells in priority i queue are k slots is

$$\frac{\delta_i}{v_i} \left(1 - \frac{\delta_i}{v_i}\right)^{k-1}$$

and the expected value of the inter-serviced time is v_i/δ_i . Because the probability that a queue is busy in a slot time is equal to the utilization of the queue, it is equal to multiplying arrival rate of cells and inter-serviced time, $\lambda(v_j/\delta_j)$. Therefore, when priority i queue is busy, the non-empty queue number is

$$v_i = \sum_{j=1, j \neq i}^{W-1} \lambda \frac{v_j}{\delta_j} + 1 \quad (10)$$

Similarly, the non-empty queue number when priority k queue is busy can be obtained. That is

$$v_k = \sum_{j=1, j \neq k}^{W-1} \lambda \frac{v_j}{\delta_j} + 1 \quad (11)$$

Subtraction of Eq. (10) from Eq. (11) yields

$$v_k = v_i - \frac{\lambda v_k}{\delta_k} + \frac{\lambda v_i}{\delta_i} \quad (12)$$

Let $p_i = \lambda v_i/\delta_i$ and $b_i = \lambda/\delta_i$, then

$$v_k = v_i - p_k + p_i \quad (13)$$

Substituting Eq. (13) into Eq. (10) one obtains

$$v_i = \sum_{j=1, j \neq i}^{W-1} b_j(v_i - p_j + p_i) + 1 \quad (14)$$

From Appendix B Eq. (14) is deduced to

$$v_i = \frac{1}{1 - \sum_{j=1, j \neq i}^{W-1} \left(\frac{\lambda}{\delta_j} + \frac{\lambda^2(\delta_j - \delta_i)}{(\lambda + \delta_j)\delta_i\delta_j} \right)} \quad (15)$$

5. Analysis of the non-equivalent case

This section presents the analytical model of the non-equivalent case for the network. The non-equivalent case is when M is equal to $k \times W$ and $k > 1$. In this case, every node has W queues because it may transmit cells to its assigned channel. This differs from the case of $k = 1$. Under the circumstance of an equivalent case, the cells of every node in one partition will defer their transmission because of that of higher priority nodes in the same partition. In the non-equivalent case, except for the above condition, the transmissions of cells of every node in one partition are also deferred because of the transmission of cells at nodes in other partitions. Section 2 revealed that every node has the priority $0, 1, \dots, W - 1$ queues. For the channel corresponding to the priority 0 queue of a node, the node is a transmitting-receiving node on the channel. A transmitting-receiving node has the best transmission opportunity in one partition because of slot reuse. Due to the identical behavior of every partition, analyzing the behavior of a partition will provide the analytical model of the network.

Like the equivalent case, the average cell delay of priority i queue can be represented as

$$T_i = TQ_i + [X_i] + D_i \quad (16)$$

and the average cell delay of the network as

$$T = \frac{\sum_{i=0}^{W-1} T_i}{W} \quad (17)$$

Herein, D_i is the propagation delay of cells in priority i queues. From the cell transmission direction, it follows that

$$D_i = \frac{\sum_{j=1}^{k-\mu(i)} (jW - i)d}{k - \mu(i)} \quad (18)$$

where $\mu(i)$, a unit-pulse function, is defined as

$$\mu(i) = \begin{cases} 0 & \text{if } i \neq 0 \\ 1 & \text{if } i = 0 \end{cases} \quad (19)$$

Because of the slot-reuse property, the behavior of transmitting–receiving nodes is more complicated than that of transmitting nodes. In a channel, the traffic rate of a transmitting–receiving node in one partition is $(k - 1) \cdot \lambda$ while the rate of transmitting nodes in one partition is $k \cdot \lambda$. Then, the rate of nodes in one partition can be represented as $(k - \mu(i))\lambda$ ($i = 0, 1, \dots, W - 1$).

Like the case of $k = 1$, the priority queue model provides the average queue-waiting delay. Here, the higher priority nodes consist of higher priority nodes in the same partition and nodes in other partitions. Then, like the deduction of Appendix A, the queue-waiting delay can be obtained

$$TQ_i = \frac{R_i}{\left(1 - \sum_{j=1}^{k-1} Wj\rho'_j - (i-1 + \mu(i))k\rho'_i + \mu(i)(k-1)\rho'_i\right)} \times \frac{1}{\left(1 - \sum_{j=1}^{k-1} Wj\rho'_j - (i-1 + \mu(i))k\rho'_i + \mu(i)(k-1)\rho'_i - (k - \mu(i))\rho_i\right)} \quad (20)$$

where $\rho'_i = \lambda'_i E[X_i]$ and $\rho_i = \lambda E[X_i]$.

From Ref. [15], it can also be calculated that

$$R_i = \frac{E[X_i^2]}{2E[X_i]} \quad (21)$$

The formulas of λ'_i , $E[X_i]$ and $E[X_i^2]$ are the same as those in the case of $k = 1$. Substitution of Eq. (21) into Eq. (20) gives an expression for TQ_i in terms of k , W , v_i , λ , $\mu(i)$

$$TQ_i = \frac{2v_i - 1}{2\left(1 - \frac{Wk(k-1)\lambda}{2} - (i-1)k\lambda - \mu(i)\lambda\right)} \times \frac{1}{2\left(1 - \frac{Wk(k-1)\lambda}{2} - (i-1)k\lambda - \mu(i)\lambda - (k - \mu(i))\lambda v_i\right)}$$

5.1. Non-empty queue number for $k > 1$

For a priority i node, the traffic rate of other partitions and higher priority nodes on the focus channel is

$$\sum_{j=1}^{k-1} Wj\lambda + (i-1)k\lambda + \mu(i)\lambda \quad (i = 0, 1, \dots, W - 1)$$

Thus,

$$\delta_i = e^{-\left(\left(\sum_{j=1}^{k-1} Wj\lambda\right) + (i-1)k\lambda + \mu(i)\lambda\right)} \quad (22)$$

Resembling the case of $k = 1$, the number of non-empty

queues can be obtained by calculating the interval at which queues are busy. Thus,

$$v_i = \sum_{j=0, j \neq i}^{W-1} (k - \mu(j))\lambda \frac{v_j}{\delta_j} + 1 \quad (23)$$

Correspondingly, the number of non-empty queues when priority k queue is busy is

$$v_k = \sum_{j=0, j \neq k}^{W-1} (k - \mu(j))\lambda \frac{v_j}{\delta_j} + 1 \quad (24)$$

Then,

$$v_k = v_i - \frac{(k - \mu(k))\lambda v_k}{\delta_k} + \frac{(k - \mu(i))\lambda v_i}{\delta_i} \quad (25)$$

Thus,

$$v_i = \sum_{j=0, j \neq i}^{W-1} \frac{(k - \mu(j))\lambda}{\delta_j} \times \left(v_i - \frac{(k - \mu(j))\lambda v_j}{\delta_j} + \frac{(k - \mu(i))\lambda v_i}{\delta_i}\right) + 1 \quad (26)$$

Like the deduction of formulas of the non-empty queue number in equivalent case, the formula of non-empty queue numbers in non-equivalent case is

$$v_i = \frac{1}{1 - \sum_{j=0, j \neq i}^{W-1} \left[\frac{(k - \mu(j))\lambda}{\delta_j} + \frac{\frac{(k - \mu(i))\lambda}{\delta_i} - \frac{(k - \mu(j))\lambda}{\delta_j}}{1 + \frac{(k - \mu(j))\lambda}{\delta_j}} \right]} \quad (27)$$

6. Numerical results

This section presents the simulated and analytical results. The CACI SIMSCRIPT II.5 simulation tool is used to simulate the network model. Here, the behavior of every node is assumed to be the same, and all channels are unidirectional and synchronized in the network. Meanwhile, the cell arrival rate of every node is the same, and the destination of all cells is assigned randomly. Therefore, cells are evenly distributed to all nodes except for their generators. The cell arrival distribution of every node is a Poisson distribution. The network has sixteen nodes and the distance between neighboring nodes is one slot time. For slotted ring networks with the destination removal policy, in stable state, the maximum throughput of a channel is twice the transmission rate of a channel when all cells are distributed evenly.

Fig. 5 presents the simulated and analytical results of average cell delay in this network. These results reveal

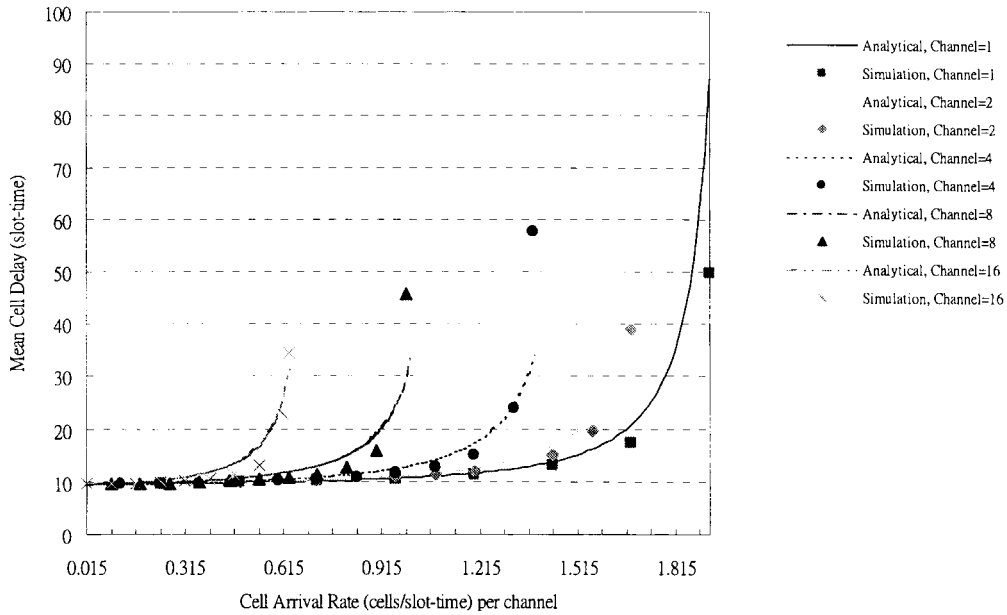


Fig. 5. Cell delay versus traffic load per channel, when the number of channels equals 1, 2, 4, 8 and 16.

that the utilization of a channel increases as nodes using this same channel as their assigned channel are increased. The reason is that the considered network with destination removal policy exploits the spatial reuse advantage and increases the utilization of the channel higher. Another phenomenon in these figures is that the throughput of the network will not be maximized. Therefore, the performance of the network can be improved when a more efficient MAC protocol is adopted. Notably, when the number of channels is one, the result is the cell delay of a single-channel slotted ring with destination removal, and one infinite cell queue at each node.

Figs. 6 and 7 illustrate the cell delay of different priority queues in this network when the number of channels equals 4 and 16, respectively. When there are 4 channels, the heaviest traffic load per channel is 1.38. Meanwhile, when there are 16 channels, the heaviest traffic load per channel is 0.63. These results indicate that when the traffic load is heavy in a channel, the cell delay of higher priority queues is smaller than that of lower priority queues. This is owing to the fact that cells of lower priority queues are harder to transmit than those of higher priority queues. This difference extends the queue-waiting delay and transmission time. This extension obviously shows the cyclic-priority property is shown in the network.

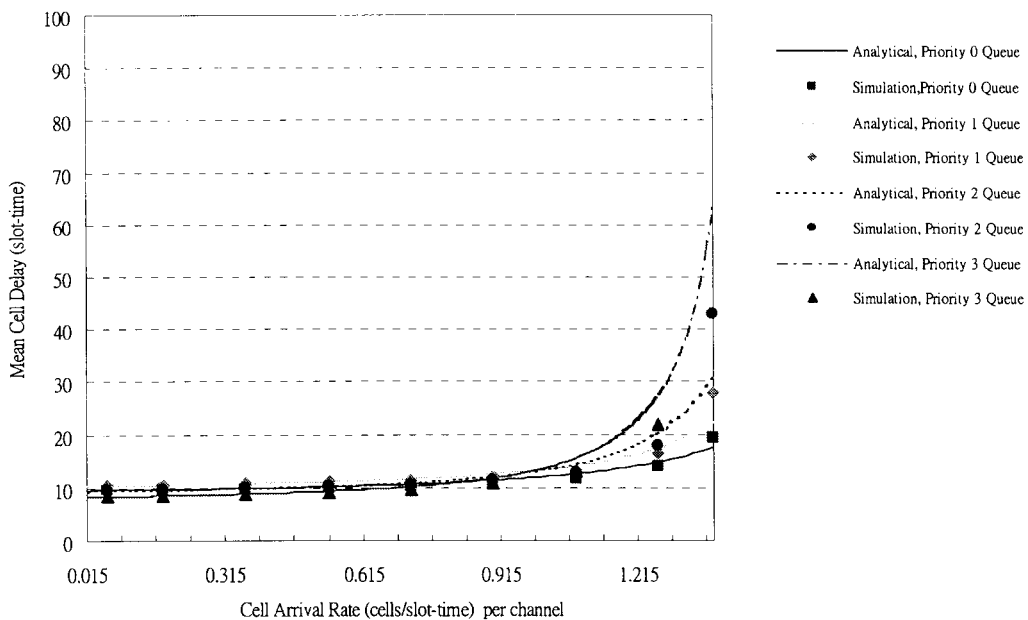


Fig. 6. Cell delay of different priority queues versus traffic load per channel, when the number of channels = 4.

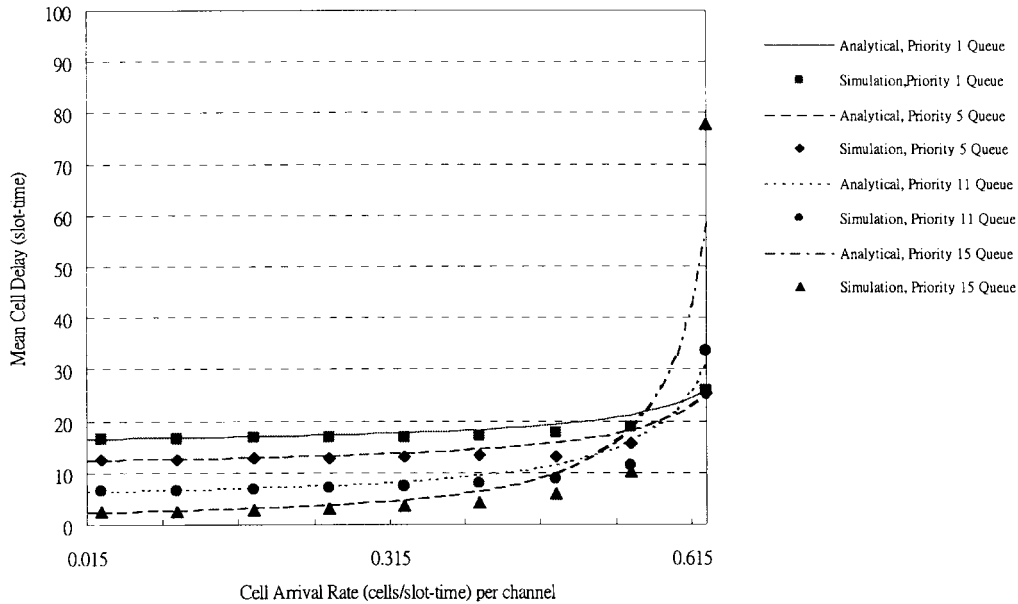


Fig. 7. Cell delay of different priority queues versus traffic load per channel, when number of channels = 16.

Fig. 8 illustrates the cell delay of cells between two nodes when the total network traffic load is 12.8. This is a simulated result. This figure reveals the symmetry in the cell delay. The queues with the same priority have the same cell delay under a balanced traffic condition. Therefore, the cell delay of the queues with the same priority in the network can be obtained only by the cell delay formula of the same priority queue in the analytical model.

7. Conclusions

This investigation describes the approximate average cell-delay analysis for multi-channel slotted ring networks with tunable transmitters and fixed receivers. It also derives the equations for the cell delay of different queues. It develops a non-preemptive priority queue model to model the behavior of the network and obtains the analytical cell-delay approximations through close-form formulas. These formulas can be modified in the future via the analytical method to produce the cell delay of queues given unbalanced traffic distribution. For verification, it applies a simulation program to obtain simulated results for the network. The simulated results closely resemble to the analytical values, and this demonstrates the performance of the network. These values can also be used as a reference for improving the network. The importance of the WDM ring network makes this research of the multi-channel slotted ring network worthwhile.

Section 6 revealed the cell delays of single-channel and multi-channel ring networks. These results indicate that network throughput increases with the number of channels, but the relationship is not proportional. Thus, in implementing the network, not all the channels need to be used for

good performance. Fewer channels imply easier implementation of the transmitter and receiver. The other characteristic of this network is the cyclic-priority property that introduces the unfairness condition. This problem could be overcome by scheduling the slots on all channels to allow all nodes fair access, or by using fairness protocols in the slotted ring network, such as Multi-MetaRing protocol [6].

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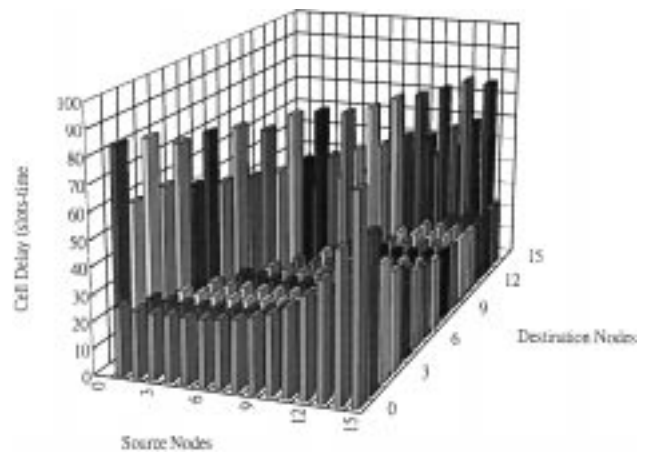


Fig. 8. Simulated cell delay between two nodes, when the number of channels = 16.

Appendix A. How to obtain the formula of the waiting delay in queue, TQ_i , in the equivalent case

Because every node can transmit a maximum of one cell in a selected interval, from Ref. [15], the cell queue-waiting delay of a highest priority node, as $i = 1$, on a channel, TQ_1 is

$$TQ_1 = R_1 + N_Q^1 E[X_1] \quad (A1)$$

Herein, N_Q^1 is the number of cells in the highest priority queue, and R_1 is the residual time seen by the node. $E[X]$ is the expected value of the random variable X .

By Little's Theorem

$$N_Q^1 = \lambda TQ_1 \quad (A2)$$

or

$$TQ_1 = R_1 + \lambda TQ_1 E[X_1] = \frac{R_1}{1 - \lambda E[X_1]} \quad (A3)$$

For the second priority node, as $i = 2$, the traffic of the highest priority node seen by the node, λ'_2 must be considered. Its expression resembles Eq. (A3) except that the additional delay due to cells of the highest priority node that arrive while a cell is queuing must be included

$$TQ_2 = R_2 + N_Q^1 E[X_2] + N_Q^2 E[X_2] + \lambda'_2 TQ_2 E[X_2] \quad (A4)$$

N_Q^1 is the queue length of the highest priority node. It differs from N_Q^1 in Eq. (A1) because the traffic rate of the highest priority node detected by the second priority node does not equal the traffic rate generated by the highest priority node. From Little's Theorem is obtained

$$TQ_2 = R_2 + \lambda'_2 TQ_2^1 E[X_2] + \lambda TQ_2 E[X_2] + \lambda'_2 TQ_2 E[X_2]$$

$$TQ_2 = \frac{R_2 + \lambda'_2 TQ_2^1 E[X_2]}{1 - \lambda_2 E[X_2] - \lambda'_2 E[X_2]} \quad (A5)$$

where TQ_2^1 is the queue-waiting delay of cells in the highest priority node for the second priority node. It is expressed in terms of R_2 , λ'_2 and $E[X_2]$. Its derivation resembles that of Eq. (A3) and

$$TQ_2^1 = R_2 / (1 - \lambda'_2 E[X_2]) \quad (A6)$$

Then the queue-waiting delay of cells in the second priority node is given by

$$TQ_2 = \frac{R_2}{(1 - \lambda'_2 E[X_2])(1 - \lambda_2 E[X_2] - \lambda E[X_2])} \quad (A7)$$

The derivation is similar for all priority cases when $i > 1$. As the cell arrival rate of every higher priority node is the same for the priority node i under assumption 4, the formula

for the queue-waiting delay is

$$TQ_i = \frac{R_i}{\left(1 - \sum_{k=1}^{i-1} \lambda'_k E[X_k]\right) \times \left(1 - \sum_{k=1}^{i-1} \lambda'_k E[X_k] - \lambda E[X_i]\right)} \quad (A8)$$

Appendix B. How to deduce formula (15) from Eq. (14) in the equivalent case

In Eq. (14), v_i is expressed by

$$v_i = \sum_{j=1, j \neq i}^{W-1} b_j (v_i - p_j + p_i) + 1 \quad (B1)$$

or

$$v_i = \left(\sum_{j=1, j \neq i}^{W-1} b_j \right) v_i - \sum_{j=1, j \neq i}^{W-1} b_j (p_j + p_i) + 1 \quad (B2)$$

By moving the right-hand part of v_i to the left-hand side, the formula is expressed by

$$\begin{aligned} \left(1 - \sum_{j=1, j \neq i}^{W-1} b_j\right) v_i &= \sum_{j=1, j \neq i}^{W-1} b_j (p_i + p_j) + 1 \\ &= \sum_{j=1, j \neq i}^{W-1} b_j (b_i v_i - b_j v_j) + 1 \\ &= \sum_{j=1, j \neq i}^{W-1} b_j (b_i v_i - b_j (v_i - p_j + p_i)) + 1 \\ &= \sum_{j=1, j \neq i}^{W-1} b_j ((b_i - b_j) v_i - b_j (p_i - p_j)) + 1 \\ &= \sum_{j=1, j \neq i}^{W-1} b_j ((b_i - b_j) v_i - b_j (b_i - b_j) v_i + b_j^2 ((b_i - b_j) v_i \\ &\quad - b_j (p_i - p_j))) + 1 \\ &= \sum_{j=1, j \neq i}^{W-1} b_j ((b_i - b_j) v_i - b_j (b_i - b_j) v_i + b_j^2 (b_i - b_j) v_i \\ &\quad - b_j^3 (p_i - p_j)) + 1 \\ &= \sum_{j=1, j \neq i}^{W-1} b_j ((b_i - b_j) v_i - b_j (b_i - b_j) v_i + b_j^2 (b_i - b_j) v_i \\ &\quad - b_j^3 (b_i - b_j) v_i + \dots) + 1 \\ &= \sum_{j=1, j \neq i}^{W-1} b_j \left(v_i (b_i - b_j) \left(\sum_{s=0}^{\infty} (-b_j)^s \right) \right) + 1 \end{aligned}$$

$$\begin{aligned}
 &= \sum_{j=1, j \neq i}^{W-1} b_j \left(v_i (b_i - b_j) \left(\frac{1}{b_j + 1} \right) \right) + 1 \\
 &= \sum_{j=1, j \neq i}^{W-1} \frac{b_j}{b_j + 1} (b_i - b_j) v_i + 1
 \end{aligned} \tag{B3}$$

or

$$\begin{aligned}
 v_i &= \frac{1}{1 - \sum_{j=1, j \neq i}^{W-1} \left(\frac{\lambda}{\delta_j} + \frac{b_j(b_i - b_j)}{b_j + 1} \right)} \\
 &= \frac{1}{1 - \sum_{j=1, j \neq i}^{W-1} \left(\frac{\lambda}{\delta_j} + \frac{\lambda^2(\delta_j - \delta_i)}{(\lambda + \delta_j)\delta_i\delta_j} \right)}
 \end{aligned} \tag{B4}$$

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